

One-loop radiative seesaw dark matter and neutrinoless double beta decay with two zero flavor neutrino mass texture

Teruyuki Kitabayashi,^{*} Shinya Ohkawa,[†] and Masaki Yasue[‡]

Department of Physics, Tokai University, 4-1-1 Kitakaname, Hiratsuka, Kanagawa, 259-1292, Japan

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We discuss the linkage between dark matter mass in the one-loop radiative seesaw model and the effective neutrino mass for the neutrino less double beta decay. This linkage, which has been already numerically suggested, is confirmed to be a reasonable relationship by deriving analytical expressions for two zero flavor neutrino mass texture.

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I. INTRODUCTION

Understanding the nature of dark matter as well as of neutrinos is one of the outstanding problems in particle physics. Recently, Ma has been proposed a simple model, so-called radiative seesaw model or scotogenic model, which can simultaneously account for the origin of neutrino masses and the presence of dark matter [1]. In this model, neutrino masses vanish at the tree level but are generated by one-loop interactions mediated by a dark matter candidate. One-loop [2] as well as two-loops [3] and three-loops [4] interactions related to neutrino mass and dark matter have been extensively studied in literature.

On the other hand, there have been various discussions on neutrino masses to ensure the appearance of the observed neutrino mixings and masses, for example, based on flavor neutrino mass matrices with two zeros [5]. This type of matrix is called the two zero flavor neutrino mass texture. If we require a nonvanishing effective neutrino mass M_{ee} for the neutrino less double beta decay [6], only four textures are compatible with observed data in the two zero flavor neutrino mass texture scheme.

In this paper, we clarify the linkage between dark matter mass in the one-loop radiative seesaw model and the effective neutrino mass for the neutrino less double beta decay. This linkage has been numerically suggested by Kubo, Ma and Suematsu [7]. Using the two zero flavor neutrino mass texture, we show this connection more explicitly by deriving analytical expressions supplemented by additional numerical calculations.

In Sec.II, we show a brief review of the radiative seesaw model and the two zero flavor neutrino mass texture. In Sec.III, we show the linkage between dark matter mass and the effective neutrino mass for the neutrino less double beta decay. Sec.IV is devoted to summary.

II. BRIEF REVIEW

A. Radiative seesaw model and dark matter

The radiative seesaw model [1] is an extension of the standard model containing three new Majorana $SU(2)_L$ singlet fermions N_k ($k = 1, 2, 3$) and one new scalar $SU(2)_L$ doublet (η^+, η^0) . These new particles are odd under exact Z_2 symmetry. Under $SU(2)_L \times U(1)_Y \times Z_2$, the main particle contents for radiative seesaw model is given by ($\alpha = e, \mu, \tau; k = 1, 2, 3$) :

$$\begin{aligned} L_\alpha &= (\nu_\alpha, \ell_\alpha) : (2, -1/2, +), \quad \ell_\alpha^C : (1, 1, +), \\ \Phi &= (\phi^+, \phi^0) : (2, 1/2, +), \\ N_k &: (1, 0, -), \quad \eta = (\eta^+, \eta^0) : (2, 1/2, -), \end{aligned} \quad (1)$$

where $(\nu_\alpha, \ell_\alpha)$ is the left-handed lepton doublet and (ϕ^+, ϕ^0) is the Higgs doublet in the standard model.

The new particles are contained in Yukawa interactions

$$\mathcal{L}_Y \supset h_{\alpha k}(\nu_\alpha \eta^0 - \ell_\alpha \eta^+) N_k + h.c., \quad (2)$$

in the Majorana mass terms

$$\mathcal{L}_N \supset \frac{1}{2} M_k N_k N_k + h.c., \quad (3)$$

and in the quartic scalar interaction potential

$$V_{int} \supset \frac{1}{2} \lambda_5 (\Phi^\dagger \eta)^2 + h.c. \quad (4)$$

Owing to the Z_2 symmetry, neutrinos remain massless at tree level but acquire masses via one-loop interactions. The neutrino flavor masses read [1]

$$M_{\alpha\beta} = \frac{\lambda_5 v^2}{8\pi^2} \sum_k \frac{h_{\alpha k} h_{\beta k} M_k}{m_0^2 - M_k^2} \left(1 - \frac{M_k^2}{m_0^2 - M_k^2} \ln \frac{m_0^2}{M_k^2} \right) \quad (5)$$

for $2\lambda_5 v^2 < m_0^2 = (m_R^2 + m_I^2)/2$ where v is vacuum expectation value of the Higgs field, m_R and m_I are the masses of $\sqrt{2}\text{Re}[\eta^0]$ and $\sqrt{2}\text{Im}[\eta^0]$, respectively.

At the one-loop level, flavor violating processes such as $\mu \rightarrow e\gamma$ are induced. The branching ratio of $\mu \rightarrow e\gamma$ is given by [7, 8]

$$\text{Br}(\mu \rightarrow e\gamma) = \frac{3\alpha_{\text{em}}}{64\pi(G_F m_0^2)^2} \left| \sum_{k=1}^3 h_{\mu k} h_{ek}^* F\left(\frac{M_k^2}{m_0^2}\right) \right|^2 \quad (6)$$

^{*} teruyuki@tokai-u.jp

[†] 6BSNM004@mail.u-tokai.ac.jp

[‡] yasue@keyaki.cc.u-tokai.ac.jp

where α_{em} denotes the fine-structure constant (electromagnetic coupling), G_F denotes the Fermi coupling constant and $F(x)$ is defined by

$$F(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x}{6(1-x)^4}. \quad (7)$$

This radiative seesaw mechanism of neutrino masses also predicts the existence of particle dark matter. The Z_2 symmetry renders the lightest Z_2 odd particle stable in the particle spectrum and this lightest Z_2 odd particle becomes a dark matter candidate. We assume that the lightest Majorana singlet fermion, which is taken to be N_1 , becomes the dark matter. For the annihilation processes $N_1 N_1 \rightarrow \ell_\alpha^+ \ell_\beta^-, \bar{\nu}_\alpha \nu_\beta$, the dark matter relic abundance can be estimated to be [7, 9]

$$\Omega h^2 \simeq 0.12 \left(\frac{0.3}{y_1} \right)^4 \left(\frac{M_1}{100 \text{ GeV}} \right)^2 \frac{(1+x)^4}{x^2(1+x^2)}, \quad (8)$$

where $h = (\text{Hubble parameter})/100$, $y_1^2 = \sum_{\alpha=e,\mu,\tau} |h_{\alpha 1}|^2$ and $x = M_1^2/m_0^2$.

B. Two zero flavor neutrino mass texture

In the two zero flavor neutrino mass texture scheme, there are 15 possible combinations of two vanishing independent elements in the flavor neutrino mass matrix. If we require a nonvanishing effective neutrino mass M_{ee} for the neutrino less double beta decay, the interesting textures are the following only four [5]

$$\begin{aligned} B_1 : & \begin{pmatrix} M_{ee} & M_{e\mu} & 0 \\ - & 0 & M_{\mu\tau} \\ - & - & M_{\tau\tau} \end{pmatrix}, \quad B_2 : \begin{pmatrix} M_{ee} & 0 & M_{e\tau} \\ - & M_{\mu\mu} & M_{\mu\tau} \\ - & - & 0 \end{pmatrix}, \\ B_3 : & \begin{pmatrix} M_{ee} & 0 & M_{e\tau} \\ - & 0 & M_{\mu\tau} \\ - & - & M_{\tau\tau} \end{pmatrix}, \quad B_4 : \begin{pmatrix} M_{ee} & M_{e\mu} & 0 \\ - & M_{\mu\mu} & M_{\mu\tau} \\ - & - & 0 \end{pmatrix}, \end{aligned} \quad (9)$$

where the mark “-” denotes a symmetric partner.

Our recent discussions [10] have found the dependence of the flavor neutrino masses on M_{ee} , which dictates, for textures labelled by $X = B_1, B_2, B_3, B_4$,

$$M_{\alpha\beta} = f_{\alpha\beta}^X(\theta_{12}, \theta_{23}, \theta_{13}, \delta) M_{ee}, \quad (10)$$

with the obvious definition of $f_{ee}^X = 1$, where $\theta_{12,23,23}$ are three neutrino mixing angles and δ is the CP-violating Dirac phase as defined in Ref.[11] and $\alpha, \beta = e, \mu, \tau$.

More explicitly, we obtain the following expressions for the B_1 texture:

$$M_{\alpha\beta} = f_{\alpha\beta}^{B_1} M_{ee}, \quad (11)$$

where

$$\begin{aligned} f_{e\tau}^{B_1} &= f_{\mu\mu}^{B_1} = 0, \\ f_{e\mu}^{B_1} &= -\frac{A_1}{c_{23}B_1 + s_{23}C_1}, \\ f_{\mu\tau}^{B_1} &= -A_1 \frac{-c_{23}B_3 + s_{23}C_3}{c_{23}B_1 + s_{23}C_1} - \frac{1 - e^{-2i\delta}}{2} \sin 2\theta_{23}, \\ f_{\tau\tau}^{B_1} &= -A_1 \frac{c_{23}B_2 + s_{23}C_2}{c_{23}B_1 + s_{23}C_1} + A_2, \end{aligned} \quad (12)$$

and

$$\begin{aligned} A_1 &= c_{23}^2 + s_{23}^2 e^{-2i\delta}, \\ A_2 &= s_{23}^2 + c_{23}^2 e^{-2i\delta}, \\ B_1 &= \frac{2c_{23}^2}{c_{13} \tan 2\theta_{12}} - t_{13} \sin 2\theta_{23} e^{-i\delta}, \\ B_2 &= \frac{2s_{23}^2}{c_{13} \tan 2\theta_{12}} + t_{13} \sin 2\theta_{23} e^{-i\delta}, \\ B_3 &= \frac{\sin 2\theta_{23}}{c_{13} \tan 2\theta_{12}} + t_{13} \cos 2\theta_{23} e^{-i\delta}, \\ C_1 &= 2 \left(\frac{s_{23}^2 e^{-i\delta}}{\tan 2\theta_{13}} - \frac{t_{13} c_{23}^2 e^{i\delta}}{2} \right), \\ C_2 &= 2 \left(\frac{c_{23}^2 e^{-i\delta}}{\tan 2\theta_{13}} - \frac{t_{13} s_{23}^2 e^{i\delta}}{2} \right), \\ C_3 &= \sin 2\theta_{23} \left(\frac{e^{-i\delta}}{\tan 2\theta_{13}} + \frac{t_{13} e^{i\delta}}{2} \right). \end{aligned} \quad (13)$$

with $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ and $t_{ij} = \tan \theta_{ij}$.

Similarly, we obtain

$$\begin{aligned} f_{e\tau}^{B_2} &= f_{\tau\tau}^{B_2} = 0, \\ f_{e\mu}^{B_2} &= \frac{A_2}{s_{23}B_2 - c_{23}C_2}, \\ f_{\mu\mu}^{B_2} &= A_2 \frac{-s_{23}B_1 + c_{23}C_1}{s_{23}B_2 - c_{23}C_2} + A_1, \\ f_{\mu\tau}^{B_2} &= A_2 \frac{s_{23}B_3 + c_{23}C_3}{s_{23}B_2 - c_{23}C_2} - \frac{1 - e^{-2i\delta}}{2} \sin 2\theta_{23}, \end{aligned} \quad (14)$$

for the B_2 texture,

$$\begin{aligned} f_{e\mu}^{B_3} &= f_{\mu\mu}^{B_3} = 0, \\ f_{e\tau}^{B_3} &= \frac{A_1}{s_{23}B_1 - c_{23}C_1}, \\ f_{\mu\tau}^{B_3} &= A_1 \frac{s_{23}B_3 + c_{23}C_3}{s_{23}B_1 - c_{23}C_1} - \frac{1 - e^{-2i\delta}}{2} \sin 2\theta_{23}, \\ f_{\tau\tau}^{B_3} &= 2A_1 \frac{(s_{23}B_3 + c_{23}C_3) \cos 2\theta_{23} - s_{23}t_{13}e^{-i\delta}}{\sin 2\theta_{23}(s_{23}B_1 - c_{23}C_1)} \\ &\quad - \frac{1 - e^{-2i\delta}}{2} \cos 2\theta_{23}, \end{aligned} \quad (15)$$

for the B_3 texture and

$$\begin{aligned} f_{e\tau}^{B4} &= f_{\tau\tau}^{B4} = 0, \\ f_{e\mu}^{B4} &= -\frac{A_2}{c_{23}B_2 + s_{23}C_2}, \\ f_{\mu\mu}^{B4} &= -A_2 \frac{c_{23}B_1 + s_{23}C_1}{c_{23}B_2 + s_{23}C_2} + A_1, \\ f_{\mu\tau}^{B4} &= -A_2 \frac{-c_{23}B_3 + s_{23}C_3}{c_{23}B_2 + s_{23}C_2} - \frac{1 - e^{-2i\delta}}{2} \sin 2\theta_{23}, \end{aligned} \quad (16)$$

for the B_4 texture.

III. RADIATIVE SEESAW DARK MATTER AND NEUTRINOLESS DOUBLE BETA DECAY

A. Analytical study

For the sake of simplicity, we assume that $N_{1,2,3}$ are nearly degenerate and we take $M_1 \sim M_2 \sim M_3 \sim M_0$ [7]. In this case, from Eqs(5) and (10), we obtain

$$\sum_k h_{\alpha k}^X h_{\beta k}^X = f_{\alpha\beta}^X \tilde{M}_{ee}, \quad (17)$$

where

$$\tilde{M}_{ee} = \frac{8\pi^2}{\lambda_5 v^2} \frac{m_0^2 - M_0^2}{M_0} \left(1 - \frac{M_0^2}{m_0^2 - M_0^2} \ln \frac{m_0^2}{M_0^2} \right)^{-1} M_{ee}. \quad (18)$$

One can readily find the following clear linkage between dark matter mass M_0 in the one-loop radiative seesaw model and the effective neutrino mass for the neutrino less double beta decay M_{ee} , which is described by

$$\begin{aligned} M_{ee} &= \frac{\sum_k h_{\alpha k}^X h_{\beta k}^X}{f_{\alpha\beta}^X} \frac{\lambda_5 v^2}{8\pi^2} \frac{M_0}{m_0^2 - M_0^2} \\ &\times \left(1 - \frac{M_0^2}{m_0^2 - M_0^2} \ln \frac{m_0^2}{M_0^2} \right). \end{aligned} \quad (19)$$

This result has been already numerically suggested by Kubo et al [7]. We confirm their suggestion by deriving analytical expressions in Eq.(19) for the two zero flavor neutrino mass texture. This is the main advantage of this paper.

As a specific example, we consider the possibility that two Yukawa couplings vanish. In the B_1 case, we obtain the following non-linear simultaneous equations

$$\begin{aligned} \sum_{k=1}^3 (h_{ek}^{B1})^2 &= \tilde{M}_{ee}, \quad \sum_{k=1}^3 h_{ek}^{B1} h_{\mu k}^{B1} = f_{e\mu}^{B1} \tilde{M}_{ee}, \\ \sum_{k=1}^3 h_{ek}^{B1} h_{\tau k}^{B1} &= 0, \quad \sum_{k=1}^3 (h_{\mu k}^{B1})^2 = 0, \\ \sum_{k=1}^3 h_{\mu k}^{B1} h_{\tau k}^{B1} &= f_{\mu\tau}^{B1} \tilde{M}_{ee}, \quad \sum_{k=1}^3 (h_{\tau k}^{B1})^2 = f_{\tau\tau}^{B1} \tilde{M}_{ee}. \end{aligned} \quad (20)$$

A solution to the non-linear simultaneous equations in Eq.(21) is obtained as

$$\begin{aligned} h_{e1}^{B1} &= \frac{(f_{e\mu}^{B1})^2 \tilde{M}_{ee} + (h_{\mu 1}^{B1})^2 \left(\frac{(f_{\mu\tau}^{B1})^2}{(f_{\mu\tau}^{B1})^2 \tilde{M}_{ee} - f_{\tau\tau}^{B1} (h_{\mu 1}^{B1})^2} + 1 \right)}{2 f_{e\mu}^{B1} h_{\mu 1}^{B1}}, \\ h_{e2}^{B1} &= -\frac{f_{\mu\tau}^{B1} \tilde{M}_{ee}}{h_{\mu 1}^{B1} \sqrt{\tilde{M}_{ee} (f_{\tau\tau}^{B1} - \frac{(f_{\mu\tau}^{B1})^2 \tilde{M}_{ee}}{(h_{\mu 1}^{B1})^2})}}, \\ h_{e3}^{B1} &= -\frac{i}{2 f_{e\mu}^{B1} h_{\mu 1}^{B1}} \left((f_{e\mu}^{B1})^2 \tilde{M}_{ee} \right. \\ &\quad \left. + (h_{\mu 1}^{B1})^2 \left(\frac{(f_{\mu\tau}^{B1})^2}{f_{\tau\tau}^{B1} (h_{\mu 1}^{B1})^2 - (f_{\mu\tau}^{B1})^2 \tilde{M}_{ee}} - 1 \right) \right), \\ h_{\mu 1}^{B1} &: \text{free parameter}, \quad h_{\mu 2}^{B1} = 0, \quad h_{\mu 3}^{B1} = i h_{\mu 1}^{B1}, \\ h_{\tau 1}^{B1} &= \frac{f_{\mu\tau}^{B1} \tilde{M}_{ee}}{h_{\mu 1}^{B1}}, \quad h_{\tau 2}^{B1} = \sqrt{\tilde{M}_{ee} (f_{\tau\tau}^{B1} - \frac{(f_{\mu\tau}^{B1})^2 \tilde{M}_{ee}}{(h_{\mu 1}^{B1})^2})}, \\ h_{\tau 3}^{B1} &= 0, \end{aligned} \quad (21)$$

where $h_{\mu 1}^{B1}$ remains as a free parameter.

Similarly, we obtain a solution in the B_2 case:

$$\begin{aligned} h_{e1}^{B2} &= f_{e\mu}^{B1}, f_{\mu\tau}^{B1}, f_{\tau\tau}^{B1}, h_{\mu 1}^{B1} \rightarrow f_{e\tau}^{B2}, f_{\mu\tau}^{B2}, f_{\mu\mu}^{B2}, h_{\tau 1}^{B2} \text{ in } h_{e1}^{B1}, \\ h_{e2}^{B2} &= f_{\mu\tau}^{B1}, f_{\tau\tau}^{B1}, h_{\mu 1}^{B1} \rightarrow f_{\mu\tau}^{B2}, f_{\mu\mu}^{B2}, h_{\tau 1}^{B2} \text{ in } h_{e2}^{B1}, \\ h_{e3}^{B2} &= f_{e\mu}^{B1}, f_{\mu\tau}^{B1}, f_{\tau\tau}^{B1}, h_{\mu 1}^{B1} \rightarrow f_{e\tau}^{B2}, f_{\mu\tau}^{B2}, f_{\mu\mu}^{B2}, h_{\tau 1}^{B2} \text{ in } h_{e3}^{B1}, \\ h_{\mu 1}^{B2} &= f_{\mu\tau}^{B1}, h_{\mu 1}^{B1} \rightarrow f_{\mu\tau}^{B2}, h_{\tau 1}^{B2} \text{ in } h_{\tau 1}^{B1}, \\ h_{\mu 2}^{B2} &= f_{\tau\tau}^{B1}, f_{\mu\tau}^{B1}, h_{\mu 1}^{B1} \rightarrow f_{\mu\mu}^{B2}, f_{\mu\tau}^{B2}, h_{\tau 1}^{B2} \text{ in } h_{\tau 2}^{B1}, \\ h_{\mu 3}^{B2} &= 0, \\ h_{\tau 1}^{B2} &: \text{free parameter}, \quad h_{\tau 2}^{B2} = 0, \quad h_{\tau 3}^{B2} = i h_{\tau 1}^{B2}, \end{aligned} \quad (22)$$

in the B_3 case:

$$\begin{aligned} h_{e1}^{B3} &= \frac{f_{e\tau}^{B3} \tilde{M}_{ee} h_{\mu 1}^{B3} - h_{e2}^{B3} \sqrt{\tilde{M}_{ee} \left(f_{\tau\tau}^{B3} - \frac{(f_{\mu\tau}^{B3})^2 \tilde{M}_{ee}}{(h_{\mu 1}^{B3})^2} \right) h_{\mu 1}^{B3}}}{f_{\mu\tau}^{B3} \tilde{M}_{ee}}, \\ h_{e2}^{B3} &= \sqrt{\tilde{M}_{ee}}, \\ h_{e3}^{B3} &= \frac{i}{f_{\mu\tau}^{B3} \tilde{M}_{ee}} \left(f_{e\tau}^{B3} \tilde{M}_{ee} h_{\mu 1}^{B3} \right. \\ &\quad \left. - h_{e2}^{B3} \sqrt{\tilde{M}_{ee} \left(f_{\tau\tau}^{B3} - \frac{(f_{\mu\tau}^{B3})^2 \tilde{M}_{ee}}{(h_{\mu 1}^{B3})^2} \right) h_{\mu 1}^{B3}} \right), \\ h_{\mu 1}^{B3} &: \text{free parameter}, \quad h_{\mu 2}^{B3} = 0, \quad h_{\mu 3}^{B3} = i h_{\mu 1}^{B3}, \\ h_{\tau 1}^{B3} &= \frac{f_{\mu\tau}^{B3} \tilde{M}_{ee}}{h_{\mu 1}^{B3}}, \quad h_{\tau 2}^{B3} = \sqrt{\tilde{M}_{ee} (f_{\tau\tau}^{B3} - \frac{(f_{\mu\tau}^{B3})^2 \tilde{M}_{ee}}{(h_{\mu 1}^{B3})^2})}, \\ h_{\tau 3}^{B3} &= 0, \end{aligned} \quad (23)$$

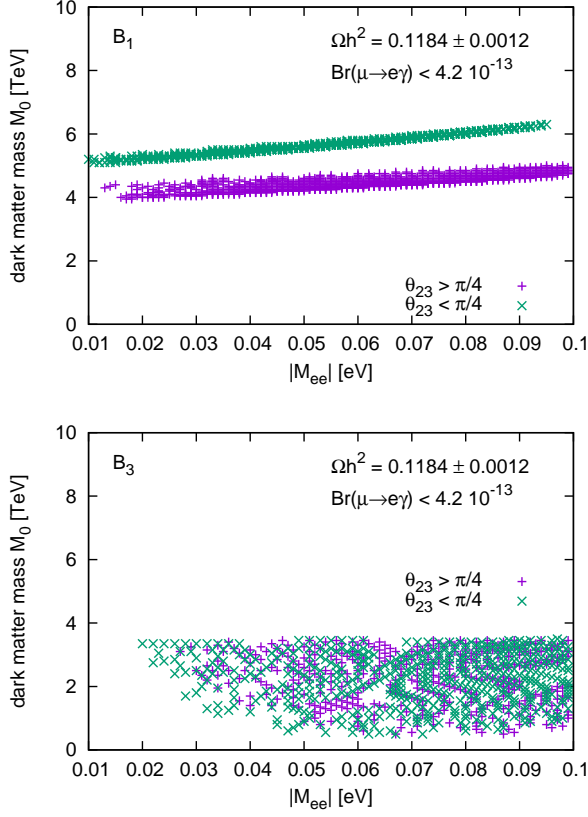


FIG. 1. Dark matter mass M_0 v.s. effective neutrino mass M_{ee} for $\Omega h^2 = 0.1184 \pm 0.0012$ [15] and $\text{Br}(\mu \rightarrow e\gamma) \leq 4.2 \times 10^{-13}$ [16]. Upper panel : B_1 case. Lower panel : B_3 case.

and in the B_4 case:

$$\begin{aligned}
 h_{e1}^{B4} &= f_{e\tau}^{B3}, f_{\mu\tau}^{B3}, f_{\tau\tau}^{B3}, h_{e2}^{B3}, h_{\mu1}^{B3} \\
 &\rightarrow f_{e\mu}^{B4}, f_{\mu\tau}^{B4}, f_{\mu\mu}^{B4}, h_{e2}^{B4}, h_{\tau1}^{B4} \text{ in } h_{e1}^{B3}, \\
 h_{e2}^{B4} &= h_{e2}^{B3}, \\
 h_{e3}^{B4} &= f_{e\tau}^{B3}, f_{\mu\tau}^{B3}, f_{\tau\tau}^{B3}, h_{e2}^{B3}, h_{\mu1}^{B3} \\
 &\rightarrow f_{e\mu}^{B4}, f_{\mu\tau}^{B4}, f_{\mu\mu}^{B4}, h_{e2}^{B4}, h_{\tau1}^{B4} \text{ in } h_{e3}^{B3}, \\
 h_{\mu1}^{B4} &= f_{\mu\tau}^{B3}, h_{\mu1}^{B3} \rightarrow f_{\mu\tau}^{B4}, h_{\tau1}^{B4} \text{ in } h_{\tau1}^{B3}, \\
 h_{\mu2}^{B4} &= f_{\mu\tau}^{B3}, f_{\tau\tau}^{B3}, h_{\mu1}^{B3} \rightarrow f_{\mu\tau}^{B4}, f_{\mu\mu}^{B4}, h_{\tau1}^{B4} \text{ in } h_{\tau2}^{B3}, \\
 h_{\mu3}^{B4} &= 0, \\
 h_{\tau1}^{B4} &: \text{ free parameter, } h_{\tau2}^{B4} = 0, \quad h_{\tau3}^{B4} = i h_{\tau1}^{B4}.
 \end{aligned} \tag{24}$$

B. Numerical calculations

Although, we have reached our main goal of this paper to analytically show the linkage between the dark matter mass and the effective neutrino mass for the two zero flavor neutrino mass texture as in Eq.(19), some additional numerical calculations may be required to visually confirm the validity of our method.

TABLE I. Dark matter mass M_0 and effective neutrino mass M_{ee} .

texture	octant	M_0 [TeV]	$ M_{ee} $ [eV]
B1	upper	3.95-5.0	0.013-0.1
	lower	5.1-5.5	0.01-0.095
B3	upper	0.5-3.45	0.027-0.1
	lower	0.5-3.5	0.02-0.1

In the neutrino sector, we use the following values of three mixing angles [12]

$$\begin{aligned}
 \sin^2 \theta_{12} &= 0.306, \\
 \sin^2 \theta_{23} &= 0.441 \text{ or } 0.587, \\
 \sin^2 \theta_{13} &= 0.0217.
 \end{aligned} \tag{25}$$

Determining the octant of θ_{23} (i.e. lower octant $\theta_{23} < 45^\circ$ or upper octant $\theta_{23} > 45^\circ$) is still an unresolved problem. For the CP-violating Dirac phase, the T2K collaboration has reported the result of their new analysis [12] and has shown that the likelihood maximum is reached at $\delta = 270^\circ$ and $\sin^2 \theta_{23} = 0.528$. We assume that

$$\delta = 270^\circ. \tag{26}$$

The remaining parameter in the neutrino sector is the effective neutrino mass. The estimated upper limit of the magnitude of the effective neutrino mass from the experiments is $|M_{ee}| \leq 0.20 - 2.5$ eV [13]. In the future experiments, a desired sensitivity $|M_{ee}| \simeq \text{a few } 10^{-2}$ eV will be reached [14]. We take

$$0.01 \leq |M_{ee}| \leq 0.1 \tag{27}$$

In the dark sector, we adopt the following standard criteria [7, 9]: (1) The quartic coupling satisfy the relation of $|\lambda_5| \ll 1$ for small neutrino masses. (2) The Yukawa couplings are sizeable. (3) The masses of new fields lie in the range between a few GeV and a few TeV. Also, we assume the additional Majorana fermion is dark matter and we require the relation of $m_0 > M_0$. We assume that

$$\begin{aligned}
 |\lambda_5| &= 1.0 \times 10^{-8}, \\
 0.01 &\leq h_{xx}^x \leq 0.1, \\
 100\text{GeV} &\leq M_0 \leq 10\text{TeV}, \\
 m_0 &= 1.5M_0.
 \end{aligned} \tag{28}$$

FIG. 1 and TABLE I show the allowed dark matter mass M_0 and effective neutrino mass M_{ee} for $\Omega h^2 = 0.1184 \pm 0.0012$ [15] and $\text{Br}(\mu \rightarrow e\gamma) \leq 4.2 \times 10^{-13}$ [16]. Although the upper limits of the branching ratio of $\text{Br}(\tau \rightarrow \mu\gamma) \leq 4.4 \times 10^{-8}$ and $\text{Br}(\tau \rightarrow e\gamma) \leq 3.3 \times 10^{-8}$ are also reported [17], we only account for $\text{Br}(\mu \rightarrow e\gamma)$ since it is the most stringent constraint. There is no allowed region of M_0 and of M_{ee} for B_2 and B_4 for the specific configuration of the Yukawa couplings shown in Eq.(22) and Eq.(24).

IV. SUMMARY

We have shown the linkage between dark matter mass M_0 in the one-loop radiative seesaw model and the effective neutrino mass M_{ee} for the neutrino less double beta decay for the two zero flavour neutrino mass texture.

The neutrino flavor masses obtained as $M_{\alpha\beta} = f(\lambda_5, h_{\alpha k}, h_{\beta k}, m_0, M_0)$ in the one-loop radiative seesaw model (Eq.(5)) and $M_{\alpha\beta} = f(\theta_{12}, \theta_{23}, \theta_{13}, \delta, M_{ee})$ for two zero flavor neutrino mass texture (Eq.(10)). The dark matter mass depends on the effective neutrino mass as follows $M_{ee} = f(\lambda_5, h_{\alpha k}, h_{\beta k}, \theta_{12}, \theta_{23}, \theta_{13}, \delta, m_0, M_0)$.

This relation has been already numerically suggested [7]. We have confirmed this result more explicitly by deriving exact analytical expression (Eq.(19)).

Numerical estimation of our analytical results is performed to show the usefulness of our proposal, We have used a specific configuration of the Yukawa couplings in the dark sector and we have assumed that two Yukawa couplings vanish such as Eq.(21). The numerical results shown in FIG. 1 and TABLE I depend on the specific configuration of the Yukawa couplings. More general analysis will be found in our future study.

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